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COSMOLOGICAL CONSIDERATIONS ON THE DIFFUSE γ-RAY ISOTROPIC BACKGROUND

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ABSTRACT

Bignami *et al.* have recently studied the problem of the origin of the diffuse γ-ray isotropic radiation. They have concluded that within standard cosmology with $\Lambda = 0$ and $p = 0$, BL Lacertae objects and Seyfert galaxies can account for most of the diffuse radiation *if* they have not evolved in time. For QSOs, an evolutionary factor $(1 + z)^4$ is allowed by the data. From the study of radio data, however, it is known that strong evolutionary effects are expected. The discrepancy cannot be explained by changing the geometry of the universe, i.e., q_0.

We present here the results of our analysis using a cosmological framework in which the gravitational constant was larger in the past, an idea that has been extensively tested in recent times. Contrary to the case of standard cosmology, it is found that in order to fit the diffuse γ-ray background, the evolutionary function required is almost identical to the one previously determined from the study of the log N–log S relation.

Subject headings: cosmology — gamma rays: general — relativity

I. INTRODUCTION

Bignami *et al.* (1979, hereafter BFHT) have investigated the contribution of Seyfert galaxies, QSOs, and BL Lacertae objects to the isotropic, diffuse γ-ray background, recently discussed by Fichtel, Simpson, and Thompson (1978). BFHT arrived at a conclusion which, if confirmed by the addition of future observational data, could be of major relevance in cosmology.

Within a Friedmann cosmology, with zero cosmological constant, the contribution of these discrete sources can account for most of the isotropic, diffuse γ-ray background between 1 and 150 MeV, provided there has been almost *no* evolution since $z \sim 4$. For QSOs, an evolutionary function up to $(1 + z)^4$ is allowed by the data.

The assertion that BL Lacertae objects, Seyferts, and to a certain extent QSOs must not have evolved during the long look-back time corresponding to $z \sim 4$, is difficult to reconcile with what the analysis of the same objects in different parts of the spectrum (optical and radio) has taught us over the years.

The word evolution is here intentionally unqualified: it could mean density and/or luminosity evolution even though the canonical evolution function $\sim(1 + z)^6$ to which we have become accustomed is cast in the form of a density evolution (Schmidt 1978). On the other hand, from the log N–log S analysis for radio sources it is not easy to separate density from luminosity evolution, and many people feel that both are actually needed to fully account for the data (von Hoerner 1973).

Should future observations confirm the results of BFHT, we would be forced to conclude that the discrete sources behave rather differently in the γ-ray region of the spectrum, and a significant astrophysical problem would ensue. The alternative of changing the cosmological model to accommodate a $(1 + z)^6$ type of evolution leaves very little room for hope; in fact, unacceptable values of H_0 and q_0 would then be required.

In this paper we shall use the cosmological framework introduced by Canuto *et al.* (1977), in which the gravitational "constant" G can vary with time as t^{-1}, to study the γ-ray diffuse background. The new scheme has already been successfully tested on (*a*) the m versus z relation for elliptical galaxies, (*b*) the isophotal angles versus z, (*c*) the metric diameters of radio galaxies and quasars, (*d*) the 3 K blackbody radiation, (*e*) the luminosity of the Sun with geophysical consequences, and finally (*f*) the log N–log S relation for radio sources (Canuto and Hsieh 1979; Canuto, Hsieh, and Owen 1979; Canuto and Owen 1979). In fitting the log N–log S relation to the radio data, we also derived an evolutionary function of the type $(1 + z)^n$, where n turned out to be somewhat smaller than its value in standard cosmology. The exponent is lower than in the standard case because of the presence of a time-varying G which can be shown to simulate the effects of source evolution.

The most important result of this paper is that contrary to the situation in standard cosmology, the evolutionary function derived from the log N–log S data fits the γ-ray diffuse background satisfactorily. This is a gratifying feature since although there is no reason to suppose that different parts of the spectrum have identical luminosity evolution, selection effects and turn-on times aside, we would expect them to have the same density evolution, and it is difficult to predict that all effects should cancel in the gamma region.

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II. THE EQUATION FOR THE INTENSITY

The mathematical details of how to construct a cosmological scheme with a varying G have been presented in detail elsewhere, and we refer to Canuto *et al.* (1977), Canuto and Hsieh (1979), Canuto, Hsieh, and Owen (1979), and Canuto and Owen (1979) for a full presentation. Here we shall present only the basic idea. Since G has dimensions, one must specify with respect to which units one has to understand its variation. Two systems of units have been introduced: gravitational and atomic. With respect to gravitational units (for instance, ephemeris time), G is a true constant and Einstein's equations retain their standard form. A new set of Einstein's equations, however, as seen by an atomic instrument, must be derived which allow G to vary. If we call Δt and Δt_E the intervals of time that it takes for a given event to occur when recorded by atomic and gravitational clocks, respectively, we shall write in general

$$\Delta t_E = \beta(t)\Delta t\,, \tag{1}$$

where $\beta(t)$ is a function of the age of the universe in atomic units. Standard cosmology postulates $\beta(t) = 1$ and therefore that the two clocks are identical at any cosmological time. The scale factors $R(t_E)$ and $R(t)$ scale as in equation (1). In performing calculations one can make use of the standard cosmological equations written in Einstein units, provided the final results, to be compared with observations (which we always make using atomic apparatus), are properly transformed to atomic units.

The basic theoretical result used by BFHT is their equation (1) expressing the intensity j in terms of the emissivity Q. The rederivation of this result in our framework is nontrivial and presented in the Appendix. The final result is

$$j = \frac{cn_0}{4\pi\bar{H}_0}\int_0^{\varkappa_*} d\varkappa\,\frac{f(z)Q[E_0(1+\varkappa)]}{(1+\varkappa)(1+2\bar{q}_0\varkappa)^{1/2}}\,\frac{G(t)}{G_0}\,. \tag{2}$$

The function is question is $f(z)$, which represents both luminosity and density evolution. For $Q(E)$, the typical source spectrum, we shall adopt the form suggested by BFHT. In equation (2) the photon intensity j is given in number of photons cm^{-2} s^{-1} keV^{-1}; $\bar{H}_0$ is the Hubble constant in gravitational units (not the one observed); $\varkappa$ is a symbol without direct physical meaning. In fact, the observed z is related to $\varkappa$ by $1 + z = \beta(1 + \varkappa)$, where $\beta(t)$ is the scale function. As in previous papers, we shall investigate the four cases

$$G\beta = 1\,, \qquad G\beta^2 = 1\,, \qquad G = \beta\,, \qquad G = \beta^2\,, \tag{3}$$

where

$$G = G_0(t_0/t) \tag{4}$$

always.

Given a value for z, the corresponding $\varkappa$ can only be found numerically for a general cosmological model. Tables of values of z and $\varkappa$ are presented in Canuto, Hsieh, and Owen 1979. There is, however, a particular case that can be solved analytically, and we think it is important to present it here so as to give a feeling for changes introduced by the variation of G.

As shown in § V*b* of Canuto, Hsieh, and Owen 1979, for the $k = 0$ case (Einstein-de Sitter in gravitational units), we have ($G\beta = 1$)

$$R(t) \sim t^{1/3}\,, \qquad (1+z)^4 = 1+\varkappa\,,$$
$$\bar{H}_0 = 4H_0\,, \qquad G \sim (1+z)^3\,, \tag{5}$$

so that equation (2) transforms to

$$j = \frac{cn_0}{4\pi H_0}\int_0^{z_*} dz f(z)Q[E_0(1+z)^4]\,, \tag{6}$$

to be compared with equation (1) of BFHT for $\bar{q}_0 = \frac{1}{2}$:

$$j(\text{BFHT}) = \frac{cn_0}{4\pi H_0}\int_0^{z_*} dz f_{\text{BFHT}}(z)\,\frac{Q[E_0(1+z)]}{(1+z)^{3/2}}\,. \tag{7}$$

It is clear that since for low energies $Q \approx E^{-a}$ ($a \geq 1$), in equation (6) we can allow

$$f(z) = f(\text{BFHT})(1+z)^{3a-3/2}\,, \tag{8}$$

or using $f(z) = (1 + z)^n$,

$$n(\text{scale cov.}) = n(\text{standard}) + 3a - 3/2\,, \tag{9}$$

and yet obtain the same answer as BFHT. For QSOs, $a = 1.4$, and therefore the evolutionary factor in (6) is larger by a factor $(1 + z)^{2.7}$, illustrating how the introduction of two scales has rendered the cosmological more sensitive to z. Using the form of Q given by BFHT, a similar argument shows that in the high-energy part of the spectrum, $f(z)$ in (6) can be $(1 + z)^{6.6}$ larger than f(BFHT).

IV. RESULTS

Using the results previously established by Canuto, Hsieh, and Owen (1979) giving $\beta(t)$ versus z and $\varkappa$ for several values of $\bar{q}_0$, we computed equation (2) using the function Q given by BFHT for Seyferts, QSOs, and BL Lacertae objects, and with $f(z) = (1 + z)^n$.

The results are shown in Figures 1–2 for the gauge $G\beta = 1$. Several comments are in order.

1. For a given n, the results depend on $\bar{q}_0$ in such a manner that the larger the value of $\bar{q}_0$, the smaller the value of j.

2. Within a closed universe ($\bar{q}_0 > \frac{1}{2}$), a value of n up to 6 is not enough to fit the data (Fig. 1).

Since we believe that $n = 6$ is the largest value we can allow (see below), we must conclude that either (*a*) the universe is not closed or (*b*) the discrete sources do not contribute significantly to the diffuse γ-ray spectrum. The two alternatives are equally tenable on the basis of this test alone. Explanations different from the one proposed by BHFT and adopted here have been put forward, as discussed by BHFT (see Stecker, Morgan, and Bredekamp 1971; Stecker 1978).

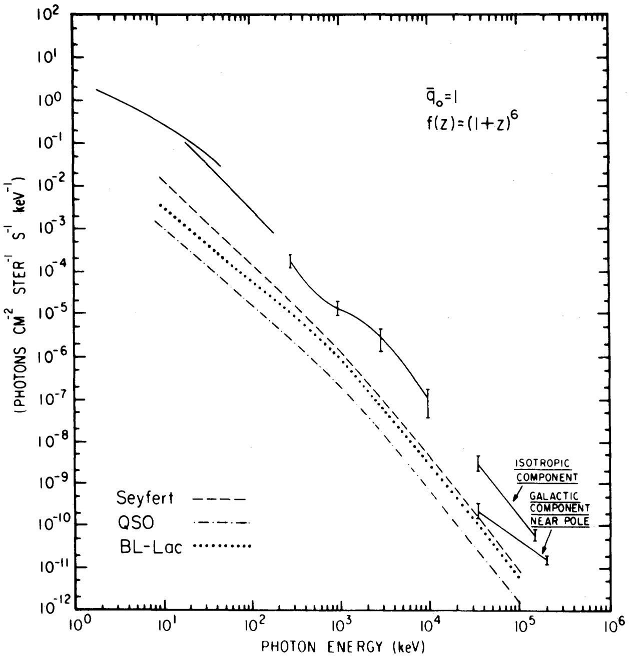

FIG. 1.—Comparison of the theoretical predictions with observations, solid lines. The data are taken from Fig. 5 of BFHT, to which the reader is referred for a complete reference for the X-ray part of the spectrum. The isotropic component and galactic component are due to Fichtel, Simpson, and Thompson (1978). Closed universe $\bar{q}_0 = 1$ and evolution $f(z) = (1 + z)^6$, $G\beta = 1$.

Should other evidence favor a closed universe, it is clear that within the present cosmological framework we would have no choice but to abandon the idea that discrete sources contribute significantly to the γ-ray diffuse background.

3. Within an open universe, $n = 0$ and $n = 6$ clearly yield the lower and upper limits for j. An intermediate value of n can therefore fit the data fairly well. For this reason, we have run a case corresponding to $n = 4$ and $\bar{q}_0 = 0.02$, the value used by BFHT. The results shown in Figure 2 fit the data satisfactorily.

4. The value of n used here is very close to that we determined previously on the basis of the log N–log S test. In that study it was found that the fit to the data seriously deteriorates as one goes from an open to a closed universe (see Figs. 4*a*–4*c* of Canuto and Owen 1979). In particular, within an open universe and $\bar{q}_0 = 0$, an evolutionary function of the type $(1 + z)^{3.5}$ yielded a satisfactory fit. (See curve 3, Fig. 4*a* of Canuto and Owen 1979; in that paper the symbol n is not to be confused with the n here).

5. Reversing the order of arguments, we find that within the present cosmological framework, consistency with the log N–log S test calls for $n \sim 4$ and therefore Figure 2.

It is found that as we change β from an increasing to a decreasing function of time, while holding all other parameters constant, the predicted values of j rise continuously. Less evolution is therefore required to meet the data as β becomes a decreasing function of time. Figures 3–5 show the theoretical results superposed on the data for $\bar{q}_0 = 0.02$ and $G\beta^2 = 1$, $G\beta^{-2} = 1$, and $G\beta^{-1.087} = 1$, where n has been

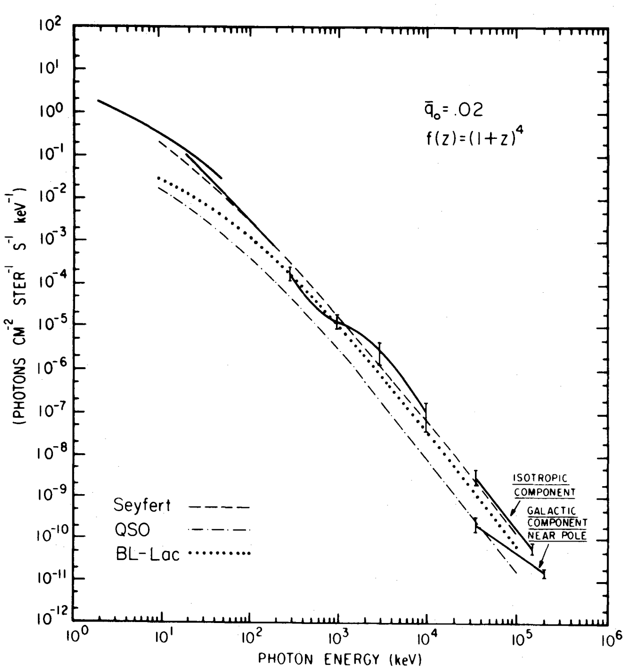

FIG. 2.—Same as Fig. 1 for an open Universe $\bar{q}_0 = 0.02$, $f(z) = (1 + z)^4$, $G\beta = 1$.

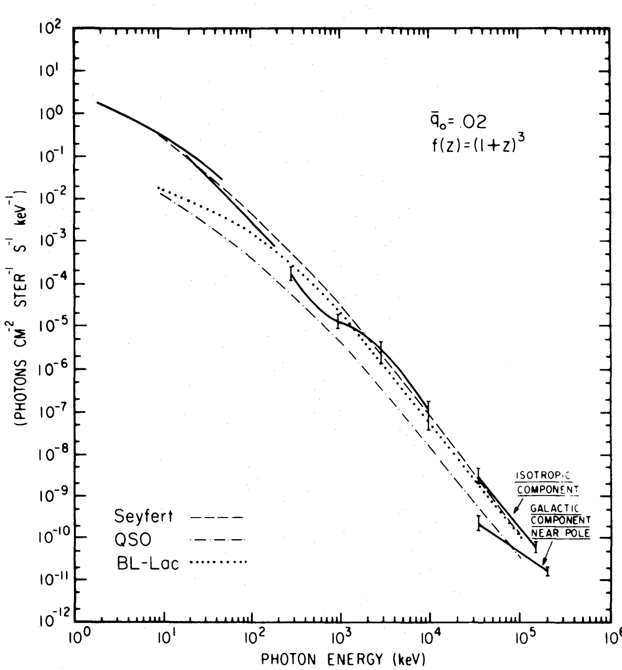

FIG. 3.—Same as Fig. 1 for an open universe, $\bar{q}_0 = 0.02$, and evolution $f(z) = (1 + z)^3$, $G\beta^2 = 1$.

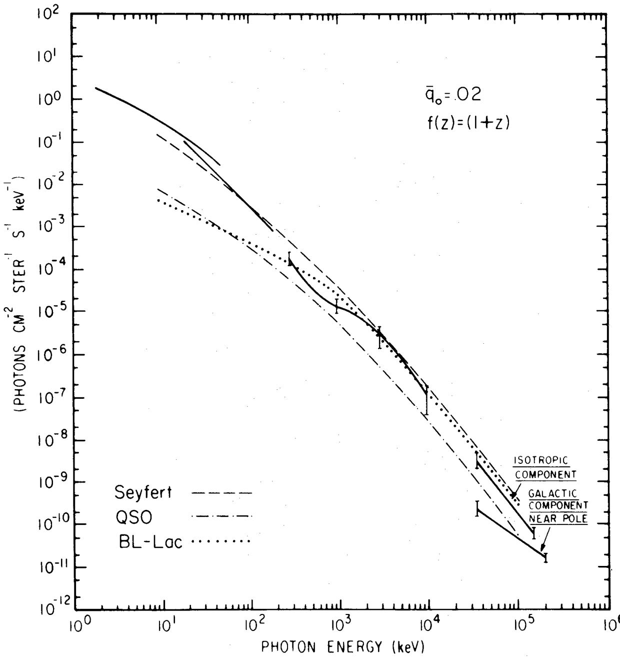

FIG. 4.—Same as Fig. 1 for an open universe, $\bar{q}_0 = 0.02$, and evolution $f(z) = (1 + z)$, $G\beta^{-2} = 1$.

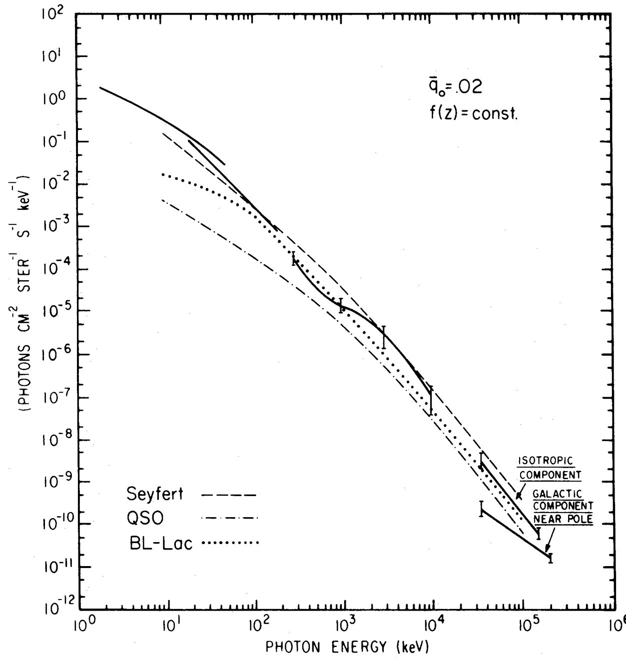

FIG. 5.—Same as Fig. 1 for an open universe, $\bar{q}_0 = 0.02$, and evolution $f(z) = $ const., $G\beta^{-1.087} = 1$.

chosen equal to 3, 1, and 0, respectively, in order to match the data.

However, it is not the particular value of n at this stage which tests the model but its consistency with n as determined by other cosmological tests. The values of n found in the present paper differ from the ones previously determined through the log N–log S test (Canuto and Owen 1979) by 1.2, 0.4, -1.4, and -1.8 (for $G\beta = 1$, $G\beta^2 = 1$, $G\beta^{-2} = 1$, and $G\beta^{-1.087} = 1$), all of which are better than the differences of 4–6 characterizing standard cosmology.

The theoretical predictions are very sensitive to $\bar{q}_0$ only for the gauge $G\beta = 1$, i.e., when β increases with time. As $\beta(t)$ decreases with time, the sensitivity becomes less pronounced to the point that for $\beta \sim 1/t$ the results are insensitive to $\bar{q}_0$ as in standard cosmology. This behavior has been found to characterize all the cosmological tests studied previously (Canuto, Hsieh, and Owen 1979; Canuto and Owen 1979).

V. CONCLUSIONS

We have throughout been effectively fitting the Seyfert contribution to the data in each cosmological case. This is because they make the largest contribution in each case, owing mainly to their large assumed number density, and the fact that on a log–log plot, addition of the QSO and BL Lacertae object contribution does not change the order-of-magnitude fit. If we wish to fit the data by making the major contribution come from these latter species, their estimated evolution would be stronger, but not as strong as their evolution at other frequencies in standard cosmology, and we would simultaneously have to lower our estimates of the evolution of the Seyferts.

One may discuss the merits of some of the astronomical assumptions which go into the model. Have BFHT, and we following them, been unfortunate in our choice of a typical spectrum $Q(E)$? Would it affect results significantly to average over a distribution of spectra? We both have assumed that the parameter a in $Q(E)$ is not evolving. We both have assumed that $z^* = 4$. Now it is certainly true that the maximum z at which QSOs have been observed is above 3 and that past statistical pronouncements about their absence beyond certain redshifts have been premature. However, the major contributors to the diffuse γ-ray spectrum seem to be Seyfert galaxies and BL Lacertae objects, and it is implicitly understood in both works that the latter exist throughout the universe out to $z^* = 4$, and if they do not appear in the surveys at that redshift, it is through a combination of selection and both density and luminosity evolutionary effects. A BL Lacertae object has been discovered at $z \sim 1.8$, and there is evidence that some objects identified as QSOs may be Seyfert's at high redshifts (Weedman 1976), supporting their hypothetical existence at this early time. The strategy is one of simplicity. However, future data may induce us to use a different value of z^* for any or all of these objects. To be prepared for this contingency, we investigated the effect of changing z^* from 4 to 1 as an example for the case of Seyfert galaxies. (And if the Seyferts do not exist beyond $z = 1$, we should fit the BL Lacertae objects or QSOs to the data as mentioned above.)

In standard cosmology it was found that for $\bar{q}_0 = 0$, when $z^* = 4$, $n = 0$ gave a reasonable fit;

when $z^* = 1$, $n = 1$ will give an equivalent fit at low energies and $n = 0.6$ at high energies. In a scale-covariant cosmology, for $\beta G = 1$ and $\bar{q}_0 = 0$, when $z^* = 4$, $n = 4$ gave a reasonable fit; when $z^* = 1$, $n = 7.8$ will give an equivalent fit at low energies and $n = 5.8$ at high energies. In general, the choice of the evolutionary exponent within the gauge $G\beta = 1$ of the scale covariant cosmology is more sensitive to the choice of z^* than in standard cosmology. In both cosmologies, however, the choice of n is more sensitive to z^* at low energies than at high energies. And if one wants to raise n, then one must lower z^*.

However, it is not the purpose of this paper to try to adapt a cosmology to the data through the judicious simultaneous choice of various parameters. After all, we can just choose our luminosity and density evolution rates to cancel in the gamma region of the spectrum. A major point of BFHT's paper is to point out the forced nature of explanations of such coincidences. It is always possible that zero is not a special number in this case. We do not expect or wish n to be identical at all wavelengths in either cosmology. It is rather the purpose of this paper to point out the sweeping effects of a scale-covariant formulation of cosmology. It is noteworthy that it is the full nature of the scale-covariant theory which *raises* our estimate of n, not the imposition of $G \sim 1/t$.

To summarize, in this test we are determining $n_\gamma = e_\gamma + s_\gamma$ and in the log N–log S test we are determining $n_R = 3/2e_R + s_R$, where e represents luminosity evolution and s density evolution. Selection effects aside, we assume that $s_\gamma = s_R$ (contrary to points 1 and 2 below). We then have $n_R - n_\gamma = 3/2e_R - e_\gamma = 1.2$, 0.4, -1.4, or -1.8 for scale covariant cosmology and 4–6 for standard cosmology. Given the uncertainties of the data and the understanding that s may be negative rather than e if one of them must be, the radio and gamma data cannot be said to contradict each other in scale-covariant cosmology.

Several caveats are applicable to standard cosmology as much as to scale-covariant cosmology. Prominent complications might be (1) gamma emission only during a particular period of the source's lifetime, which would lead us to overestimate the contributions to the diffuse background; (2) gamma emission only by a subclass of the sources considered, which would lead us to overestimate the number of contributing sources; and finally (3) gamma emission in a beam which would lead us to overestimate the intrinsic luminosity.[3] Points (1) and (2) can apply to radio sources as a subclass of gamma sources as much as to gamma sources as a subclass of radio sources.

These three effects could be leading us to underestimate n in this test. The factors above would have to be of the order of about two decades in order to cover the discrepancy of 4–6 found in standard cosmology. On the other hand, the agreement obtained in scale-covariant cosmology without making any particular assumptions about the life history of the sources is remarkable.

[3] We are particularly in debt to Dr. G. Bignami for succinctly enumerating these points.

APPENDIX

DERIVATION OF EQUATION (2)

From the work of Canuto, Hsieh, and Owen (1979, eq. [7.4b]), it is known that the relation between the apparent, l, and absolute luminosity, L, is given by

$$l = \frac{L(t)}{4\pi d_L^2} \frac{G(t)}{G_0}, \quad d_L = r_e R_0(1 + \varkappa) . \tag{A.1}$$

As discussed in the paper quoted, great care must be exercised in differentiating between z and $\varkappa$, which are related by

$$1 + z = \beta(1 + \varkappa) , \quad \beta = \beta(t) = \beta(z) . \tag{A.2}$$

Let us now introduce two quantities $B(E)$ and $I(E_0)$ with the following dimensions:

$$[B(E)] = \frac{\text{ergs}}{\text{s (Mpc)}^3} \frac{1}{\text{keV}} ; \qquad [I(E_0)] = \frac{\text{ergs}}{\text{s cm}^2} \frac{1}{\text{keV}} . \tag{A.3}$$

As proved in Canuto and Hsieh (1979, eqs. [3.14] and [3.15]), we have

$$E = E_0(1 + z)\beta^{-1} = E_0(1 + \varkappa) . \tag{A.4}$$

Clearly $\Delta E_0 I(E_0)$ has the units of l and $\Delta E B(E) dV$ has the units of L, where dV is the volume.

Using equation (A.1), we shall therefore have

$$I(E_0) = \frac{1}{4\pi} \int \frac{\Delta E}{\Delta E_0} \frac{G(t)}{G_0} \frac{B(E)}{4\pi d_L^2} dV(z) = \frac{1}{4\pi} \int \frac{G(t)}{G_0} \frac{(1 + \varkappa)B(E)}{4\pi d_L^2} dV(z) . \tag{A.5}$$

To be more explicit, we shall write

$$B(E) = B[E_0(1 + \varkappa), z] , \tag{A.6}$$

where the extra dependence upon the atomic z comprises possible luminosity and/or density evolution. The quantity dV must now be computed. We obtain

$$dV = -4\pi\left[\frac{R(t)}{R_0}\right]^3 \frac{cdt}{R(t)/R_0} \frac{d_L^2}{(1 + \varkappa)^2} . \tag{A.7}$$

The quantity $dt/R(t)$ can now be transformed. In fact, it is equal to $\beta dt/R(t)\beta(t) = dt_E/R(t_E)$, where $R(t_E)$ satisfies the standard Einstein equations, and so finally

$$dV = 4\pi\left(\frac{c}{\bar{H}_0}\right)\left[\frac{R(t)}{R_0}\right]^3 \frac{d_L^2}{(1 + \varkappa)^3} \frac{d\varkappa}{(1 + 2\bar{q}_0\varkappa)^{1/2}} . \tag{A.8}$$

The quantity $[R(t)/R_0]^3 = (1 + z)^{-3}$ is written in atomic units, whereas the remainder of equation (A.8) is expressed in gravitational units. Substituting (A.8) into (A.5), we finally have

$$I(E_0) = \frac{c}{4\pi\bar{H}_0} \int_0^{\varkappa_*} \frac{d\varkappa}{(1 + z)^3} \frac{B[E_0(1 + \varkappa), z]}{(1 + \varkappa)^2(1 + 2\bar{q}_0\varkappa)^{1/2}} \frac{G(t)}{G_0} . \tag{A.9}$$

However similar (A.9) might look to the standard expression (Schwartz 1978; Avni 1978, see eq. [3]), there are marked differences not only because of the presence of $G(t)$, but also because of $\bar{H} \neq H$ and $z \neq \varkappa$. Since the physical redshift is z and not $\varkappa$, the upper limit $\varkappa_*$ cannot be identified with the value 4–5 usually employed in standard cosmology. In fact, since $\beta(1 + \varkappa) = 1 + z$, $\varkappa$ can be significantly different (either larger or smaller) from the corresponding z. Finally, the gravitational constant $G(t)$ depends upon the atomic time and not on the gravitational time t_E.

To reduce (A.9) to a form useful for our future analysis, we shall define

$$I(E_0) \equiv j(E_0)\Delta E_0 \tag{A.10}$$

and

$$\begin{aligned} B(E, z) &= n(z)\Delta E Q(E, z) \\ &= n_0(1 + z)^3\Delta E f(z) Q(E) , \end{aligned} \tag{A.11}$$

where $Q(E)$ is the emissivity per galaxy per energy interval, and $f(z)$ accounts for evolutionary effects. With these definitions, we finally obtain, using (A.4),

$$j(E_0) = \frac{cn_0}{4\pi\bar{H}_0} \int_0^{\varkappa_*} \frac{d\varkappa f(z)}{(1 + \varkappa)(1 + 2\bar{q}_0\varkappa)^{1/2}} Q[E_0(1 + \varkappa)] \frac{G(t)}{G_0} , \tag{A.12}$$

which is the general expression valid for any gauge, i.e., any relation between β and $G(t)$. The form of $G(t)$ is also totally general at this stage.

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